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An improved model-based predictive control of vehicle trajectory by using nonlinear function †

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Abstract

A new model-based predictive control algorithm for vehicle trajectory control is proposed by using vehicle velocity and sideslip angle. Based on the error function combined with vehicle velocity and side slip of a bicycle model, a predictive control method has been proven to be useful on low velocity. Thus, it could be applied for an autonomous vehicle without a driver. Although an autonomous robot is not necessary to be driven with a high velocity, a commercial vehicle has to be driven at high velocity. Thus the previous predictive control formulation is not enough for a commercial driving system. This study is proposed to enhance the capacity of the predictive controller for rather high speed vehicles.

Keywords: Model-based predictive control; Vehicle trajectory tracking; Multibody simulation; Vehicle dynamics; Nonlinear steering function

1. Introduction

Since the trajectory control of a vehicle has many applications, such as unmanned ground vehicle, autonomous robot, military equipment, and auto-parking system, nonlinear control has become one of the key issues in the control area. Recently, several control logics were proposed for the control system of an autonomous vehicle system. Morin [1] studied several types of feedback controllers in vehicle trajectory tracking. He suggested stability and convergence limitation of feedback controllers, which were derived from linearized dynamic equations. To overcome the linearization problem, several modern control algorithms were proposed. The MPC (Modal-based predictive controller) proposed by Ollero [2] was one of the most widely used algorithms in mobile unicycle robots. Gu [3] proposed a neural predictive controller (NPC) for a car-like robot. Since the NPC uses a neural network model, a learning process is required for optimization. The most recently developed MPC was proposed by Lee [4], in which vehicle dynamic equations were directly used. This method shows an easy application in controller design because it does not require an optimization process. This basic MPC algorithm uses a mathematical method to linearize nonlinear functions that consist of side slip angle and vehicle velocity.

The basic MPC algorithm by Lee [4] has shown a good stability. Its capability, however, is limited to a low velocity application because the control law is based on the simple kinematic constraint equation. In general, vehicle dynamic equations are too complex to be directly used for controller design, so most classical vehicle controllers used linearized kinematic formulations. But, in the basic MPC model, the nonlinear formulations are used outside of control block with substituted coupled vectors. So, it is possible to apply nonlinear equations to the reduction process of control vectors.

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In this paper, a nonlinear function is used for calculation of steering angle. The nonlinear function considers not only the side-slip angle but also the vehicle velocity. So the new MPC model proposed in this paper has better performance in the high velocity condition.

2. EOM of vehicle system

In general, dynamic vehicle equations are expressed in planer dynamic equation. The bicycle dynamic model is shown in Fig. 1. The tire forces are acting on wheel center and the equation of motion can be simply derived as (1).

$$m \cdot \upsilon \cdot (\dot{\psi} + \dot{\beta}) = F_{yf} + F_{yr} \tag{1}$$

F_{yf} and F_{rf} mean lateral forces in front and rear tires that are expressed in the chassis frame. Lateral forces are a function of cornering stiffness C and lateral slip angle α, which can be described as a function of steering angle δ_f, side-slip angle β, yaw velocity $\dot{\psi}$, distance from front wheel to center of vehicle l_f, and vehicle velocity υ as shown in (2).

$$F_{yf} = C_f \alpha_f \,,\; F_{yr} = C_r \alpha_r$$
$$\text{where} \quad \alpha_f = \delta_f - \beta - \frac{\dot{\psi} \cdot l_f}{\upsilon} \,,\; \alpha_r = -\beta + \frac{\dot{\psi} \cdot l_r}{\upsilon} \tag{2}$$

In the simple kinematic formulation, yaw velocity can be derived from geometric relation as shown (3).

$$\dot{\psi} = \frac{1}{R}\upsilon = \frac{1}{l_r}\upsilon \sin\beta \qquad \left(R = \frac{l_r}{\sin\beta} \right) \tag{3}$$

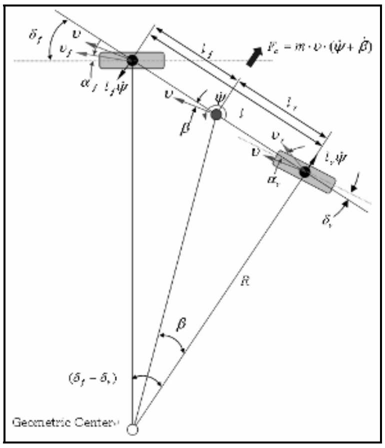

Fig. 1. Bicycle dynamic model.

Finally, using (1) to (3), the steering angle function is derived as (4). In the basic predictive controller [4], the steering angle function was defined as only a function of side slip angle β, which did not include velocity effect. But, in the new function proposed in this paper, velocity effects are also included.

There are three state variables shown in (4). Vehicle velocity and side slip angle are calculated from predictive controller, but sideslip velocity $\dot{\beta}$ is not included in the control variables. Since $\dot{\beta}$ is not included in control values, we should determine or neglect the $\dot{\beta}$ term. Since the coefficient of $\dot{\beta}$ in (4) is a small value because of a large value of C_f, the $\dot{\beta}$ term was neglected in this research.

$$\delta_f = \left(\frac{m \cdot \upsilon^2}{C_f l_r} + \frac{l_f}{l_r} - \frac{C_r}{C_f} \right) \sin\beta + m \frac{\dot{\beta}}{C_f} \upsilon + \left(1 + \frac{C_r}{C_f} \right) \beta \tag{4}$$

3. Controller design

The model-based predictive controller is based on the error function. A tracking error function is generally defined by a vector between the predictive reference vector and a controlled vehicle traveling vector as shown in Fig. 2. The error functions are a velocity equation that is calculated by current error distance and velocity, which is derived from the velocity equation $(\upsilon = r\omega)$.

Eq. (5) shows error functions of vehicle trajectory that consists of side slip angle and velocity of vehicle. The main idea of a predictive control is generation of nonlinear feed-forward functions and linearization feed-back vector. In this equation, there are too many nonlinear terms of state variables to generate feed-forward term. Lee [4] solved this problem by using a coupled vector, which was a substitution method of linearization a trigonometric function to linear vector.

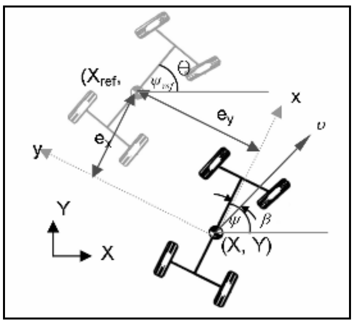

Fig. 2. Error vector in trajectory tracking.

It was very effective for extracting a linear vector formulation from the composed control vector.

$$\dot{e}_x = \upsilon_{ref} \cdot \cos e_\psi - \upsilon \cdot \cos\beta + e_y \cdot \frac{\sin\beta}{l_r}\upsilon$$
$$\dot{e}_y = \upsilon_{ref} \cdot \sin e_\psi - \upsilon \cdot \sin\beta - e_x \cdot \frac{\sin\beta}{l_r}\upsilon \quad (5)$$

The coupled vectors are separated into feed-forward and feed-back vector, which are shown in (6) and (7). The separation processes are well described in reference [4]. Vectors $\overline{x}$ and $\overline{y}$ mean coupled vectors and υ_{ref} means reference vehicle velocity. The reference vehicle path and velocity are plant input values.

$$\begin{pmatrix} \overline{x}_f = \upsilon_{ref} \cdot \cos e_\psi \\ \overline{y}_f = \dfrac{\upsilon_{ref} \cdot \sin e_\psi}{\kappa} \end{pmatrix} \quad (6)$$

Feed-back vectors can be expressed as state space functions that consist of feed-forward vectors and feed-back state variables as shown in (7).

$$\dot{e} = A(e) + B(e)u_b$$
$$\text{where } A = \begin{bmatrix} \dfrac{e_y}{l_r}\overline{y}_f \\ 0 \end{bmatrix},\ B = \begin{bmatrix} -1 & \dfrac{e_y}{l_r} \\ 0 & -\kappa \end{bmatrix},\ u_b = \begin{bmatrix} \overline{x}_b \\ \overline{y}_b \end{bmatrix}, \quad (7)$$
$$\kappa = \left(1 + \frac{e_x}{l_r}\right)$$

Eq. (7) is a state space form of feed-back function. The idea of MPC is to minimize the differences between the predicted vehicle-trajectory error and the reference robot trajectory error in a certain predicted interval. So Eq. (7) is converted into a quadratic cost function and the feed-back control vector, which is derived by a partial differential equation with a quadratic cost function [5].

Two kinds of control vectors are summed and translated into original control vectors (β, υ). Finally, the steering angle is calculated from (4). Input velocity υ is directly sent to the vehicle system and that is controlled by a simple PD controller. The total process in the MATLAB Simulink model is shown in Fig. 3.

The main idea of this paper was to change the calculation method of the steering angle. By using nonlinear dynamic equations of sideslip angle and velocity, a high performance of controller was obtained.

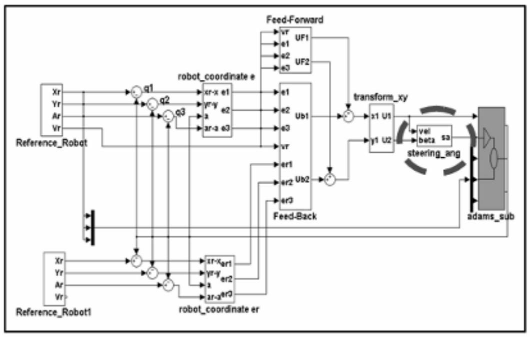

Fig. 3. Matlab simulink model of controller.

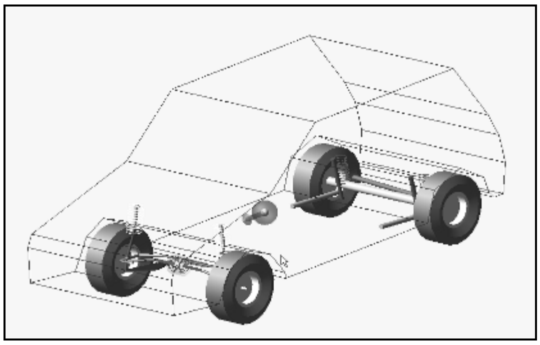

Fig. 4. Multibody dynamics vehicle model.

4. Simulation

In this paper, to test for the performance of the proposed controller, MBD (multibody dynamic) simulation technique is used. MBD simulation is an effective method for testing nonlinear mechanical systems, especially, non-holonomic conditions are included in the system. MBD simulation includes various modeling schemes to include nonlinear subsystems such as tire, bushing, contact and geometric constraints. Tires are typical nonlinear and non-holonomic objects. To solve this difficult problem, many tire models were developed for MBD simulation. The MF-tire (magic formula tire) is one of the most widely used models for handling simulation in the vehicle industry. The ADAMS program is also certified for vehicle simulation; thus those are employed in this research.

The MBD vehicle system used in this research is shown in Fig. 4. This vehicle is a compact car model produced in Korea [6].

Table 1. Vehicle property & controller parameters.

Vehicle Specification		Controller Parameter	
Total Mass	700 kg	lf	1335 mm
Wheel Base	2335 mm	lr	1000 mm
Tread	1260 mm	Cf	36000
Front Susp.	Macpherson	Cr	36000
Rear Susp.	Trailing Axle		
Tire Model	MF-Tire Model		

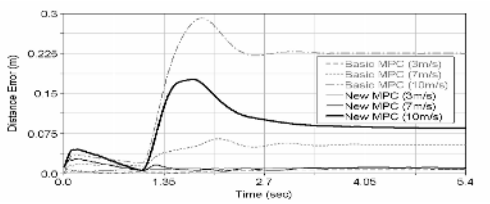

Fig. 5. CRC results: distance error (basic vs. new).

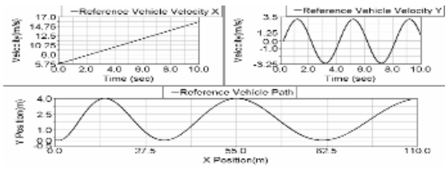

Fig. 6. Sine sweep: driving path & velocity.

This vehicle model possesses 15 degrees of freedom (DOF), which includes 6 DOF in the chassis, 8 in the suspension and wheel rotation, and 1 for the steering. The vehicle specifications are summarized in Table 1.

Controller parameters are set by vehicle data and cornering stiffness is defined from general property of tire [7], which is also shown in Table 1. The same predictive controller weighting factors in reference [4] are also used in the paper.

5. Simulation results

To test performance of the proposed controller, CRC (constant radius cornering) simulations are performed with different velocities (3, 7, 10 m/s) as shown in Fig. 5. In the figure, ‘New’ and ‘Basic’ mean the proposed method and the basic MPC [4], respectively. Fig. 5 shows magnitudes, which are RMS (root mean square) errors of x and y distances.

Sine sweep tracking simulation was carried out to proof high velocity tracking capability of the proposed controller. The input velocity along X direction is increased with a constant acceleration and Y velocity is set to a sinusoidal function as shown Fig. 6. The reference vehicle velocity increased to almost 16 m/s.

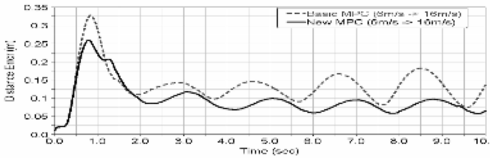

Fig. 7. Sine sweep: distance error (basic vs. new).

The new proposed controller shows better convergence and maintains controllability against increscent of velocity as shown in Fig. 7.

6. Conclusion

The new MPC control algorithm was proposed to enhance tracking capability with high speed maneuvering. In the proposed MPC control algorithm, vehicle velocity was additionally included in the steering angle function. Thus, better performance than the basic MPC algorithm was obtained. It might be very useful for driving control of a vehicle. But the current state of the proposed MPC is still not enough to control very high velocity vehicles (over 80 km/h) because the proposed MPC might diverge on that high velocity. Since the inertia of a vehicle is affected at high velocity condition, roll angle and lateral acceleration should also be included for high velocity maneuvering. These will be further topics to improve the proposed MPC algorithm.

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Dr. Wan-Suk Yoo received his Ph.D. degree in 1985 from the University of Iowa. In 1994, he became a full professor at the Pusan National University, and he was selected an ASME fellow. He is serving as a vice-president of the KSME.